MetaOCaml Theory and Implementation

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Abstract

Quasi-quotation (or, code templates) has long been used as a convenient tool for code generation, commonly implemented as a pre-processing/translation into code-generation combinators. The original MetaOCaml was also based on such translation, done post type checking. BER MetaOCaml employs a significantly different, efficient (especially in version N114) translation integrated with type-checking, in the least intrusive way. This paper presents the integrated efficient translation for the first time.

1 Introduction

(BER) MetaOCaml [4, 5] is a superset of OCaml to generate assuredly well-formed, well-scoped and well-typed code using code templates, also known as brackets and escapes. For example:

let eta = **fun** f \rightarrow .<**fun** x \rightarrow .~(f .<x>.)>. (* val eta : (α code \rightarrow β code) \rightarrow ($\alpha \rightarrow \beta$) code = <fun> *)

Although the function looks banal, it has a long history and special significance in partial evaluation, where it is called 'the trick' [3]. Brackets .<...>. enclose code to generate: in our case, the code of a function. An escape .~ marks a hole in the template; the escaped expression is to generate the code to plug into the hole. Brackets are akin to string quotation marks '": indeed, a template without holes can be converted to a string and written into a file. Unlike strings, however, code templates have structure: the code within a template has to be well-formed – moreover, well-typed OCaml code. If that code has type α , the whole template has the type α code. Code templates without holes are values and can be passed as arguments (as seen in f .<x>.) and returned as function results. Templates may contain open code, such as .<x>., which is literally the code of a free variable. Here is an example of using eta, with the detailed reduction sequence:

eta (fun z
$$\rightarrow$$
 .<4 * 5 * .~z>.)
 $\rightarrow_{\beta v}$.\rightarrow .~((fun z \rightarrow .<4 * 5 * .~z>.) ..)>.
 $\rightarrow_{\beta v}$.\rightarrow .~(.<4 * 5 * .~(..)>.)>.
 \rightarrow_{splice} .\rightarrow .~(.<4 * 5 * x>.)>.
 \rightarrow_{splice} .\rightarrow 4 * 5 * x>.

The 'quoted' (i.e., templated) code remains as is: for example, 4 * 5 is not reduced. The evaluation – substitution of values for bound variables $\rightsquigarrow_{\beta v}$ and filling-in a hole in the template with the bracketed value $\rightsquigarrow_{splice}$ – occurs either outside of brackets or within escapes. If we enter eta (**fun** z \rightarrow .<4 * 5 * .~z>.) at the MetaOCaml top-level, we indeed see

 $-: (int \rightarrow int) \text{ code} = . < \mathbf{fun} x_1 \rightarrow 4 * 5 * x_1 >.$

The bound variables get automatically renamed: looking a bit ahead, choosing fresh variable names is the responsibility of the mkl code combinator, Fig. 3.

A template without holes and free variables such as above (socalled close code value) is the generated code: it can be written into a file and compiled, and even linked back into the generated program and invoked there. The function Runcode.run provided by MetaOCaml does the compilation-linking steps:

```
let g = Runcode.run .<fun x_1 \rightarrow 4 * 5 * x_1>.;;
(* val g : int \rightarrow int = <fun> *)
g 3;;
(* - : int = 60 *)
```

Thus the product 4 * 5 is computed only when the generated code is compiled and then executed – at a later, future stage, so to speak.

(One may get an inkling why eta is called 'the trick'.) MetaOCaml is hence a multi-staged language.

This paper presents the theory of BER MetaOCaml implementation. It uses the standard in theoretical CS mathematical notation and looks theoretical. The notation, however, is the *pseudo-code of the actual implementation*. The paper is written to prototype in the mathematical notation the new efficient translation, §3.1, and clarify its subtle points. It is incorporated into the recently released (May 2023) version N114 of BER MetaOCaml. The characteristic and surprising feature of the translation is using what feels like only two stages to support multiple.

The author could not believe that this is correct, and hence this paper was written to convince him. The implementation in BER MetaOCaml N114 was then done by literally transcribing the pseudo-code of Fig. 4 into OCaml. It worked the first time, passing all tests in the extensive MetaOCaml testing suite.

2 Type-checking staged programs

Variables	f, x, y, z
Types	$t ::= int \mid t \to t$
Integer constants	$i ::= 0, 1, \ldots$
Expressions	$e ::= i \mid x \mid e \mid \lambda x. e$
Environment	$\Gamma ::= \cdot \mid \Gamma, x:t$

Figure 1. Base calculus: simply-typed lambda calculus with integers

We start with the base calculus: it is the utterly standard simply typed lambda calculus with integers, shown merely for the sake of notation, particularly the notation of the typing judgment: $\Gamma \vdash e \Rightarrow e : t$. The notation makes it explicit that type checking is type reconstruction: converting an 'untyped' expression *e* to the type-annotated form e : t -or, in terms of the OCaml type checker, converting from Parsetree to Typedtree. §3.1 shows a non-trivial use of this notation.

$$\frac{1}{\Gamma \vdash i \Rightarrow i: \text{ int }} \frac{x: t \in \Gamma}{\Gamma \vdash x \Rightarrow x: t} \frac{\Gamma \vdash e \Rightarrow e: t' \to t \quad \Gamma \vdash e' \Rightarrow e': t'}{\Gamma \vdash e e' \Rightarrow (e: (t' \to t) e': t'): t}$$

$$\frac{1}{\Gamma \vdash \lambda x. e \Rightarrow (\lambda x: t'. e: t): (t' \to t)}$$

We assume that the initial environment Γ_i to type check the whole program contains the bindings of standard library functions such as succ, addition, etc. In the rule for abstraction, one may wonder where does the type t' come from. For the purpose of the present paper, one may consider it a 'guess'. After all, our subject is not type inference, but staging – to which we now turn.

Figure 2 presents the staged calculus: the Base calculus extended with bracket <*e*> and escape ~*e* expression forms and code types <*t*>.

Variables	f, x, y, z
Types	$t ::= int \mid t \to t \mid \langle t \rangle$
Integer constants	$i ::= 0, 1, \ldots$
Expressions	$e ::= i \mid x \mid e \mid e \mid \lambda x. \mid e \mid \mid \sim e$
Stage	$n,m \ge 0$
Environment	$\Gamma ::= \cdot \mid \Gamma, x^n : t$

Figure 2. Staged calculus

The calculus is actually *multi-staged*: brackets may nest arbitrarily, e.g., $\langle 1 \rangle \rangle$. The level of nesting is called *stage*. The present stage, stage 0, is outside of any brackets. An expression at stage 1 or higher is called future-stage. As should be clear from the eta example in §1, the evaluation only happens at the present stage. The typing judgment $\Gamma \vdash_n e \Rightarrow e : t$ is now annotated with stage $n \geq 0$. All variable bindings in Γ are also annotated with their stage: $x^n : t$.

The rules for integer constants and application remain the same, modulo replacing \vdash with \vdash_n : in general, most typing rules are unaffected by (or, are invariant of) staging. This is a good news for implementation: adding staging to an extant language does not affect the type checker to large extent. Here are the changed and new rules:

$x^m:t\in\Gamma$	$\Gamma, x^n : t' \vdash_n e \Longrightarrow e : t$
$\frac{1}{\Gamma \vdash_n x \Longrightarrow x^m : t} m \le n$	$\overline{\Gamma \vdash_n \lambda x. e \Rightarrow (\lambda x^n : t'. e : t) : (t' \to t)}$
$\Gamma \vdash_{n+1} e \Longrightarrow e : t$	$\Gamma \vdash_n e \Rightarrow e : $
$\overline{\Gamma \vdash_n <\!\!e\!\!> \Rightarrow <\!\!e:t\!\!>:<\!\!t\!\!>}$	$\overline{\Gamma \vdash_{n+1} {\sim} e \Rightarrow {\sim} (e:{<}t{>}):t}$

The type-checker also annotates variable references with the stage, in addition to the type. A variable bound at stage *n* may be used at the same stage – or higher (but not lower!). A present-stage variable may appear within brackets: so-called *cross-stage persistence* (or, CSP). As one may expect, bracket increments the stage for its containing expression and escape decrements. Furthermore, escapes must appear within a bracket.

For example, <<~(<1>)>> has the type <<int>>, the expression << λx . ~(f x)>> is ill-typed but << λx . ~(f < x>)>> is well-typed in an environment where f is bound to a function <int> \rightarrow <int> at stage 0. §1 has more examples.

3 Translating brackets and escapes away

After a program is type-checked and converted to the type-annotated form (a.k.a., Typedtree), we have to compile it. The type-annotated form contains brackets and escapes, so our compilation has to account for them. One popular approach [1, 2] is to post-process the type-annotated expression to eliminate all brackets and escapes. The post-processed Typedtree then has the same form as in the ordinary OCaml; therefore, we can use the OCaml back-end (optimizer and code generator) as it is – which is what MetaOCaml does.

Formally, the result of post-processing is the Base calculus enriched with code types (as well as string types and literals) and whose initial environment contains the functions in Fig. 3. We call this calculus Base₁.

The post-processing is actually a family of translations: [e] and $[e]_n^1$, which take a type-annotated expression e : t of Staged calculus and produce the type-annotated Base₁ calculus expression e':

$$\lceil e:t \rceil = e':t \qquad \lceil e:t \rceil_n^1 = e': \langle t \rangle \tag{1}$$

The expression e: t in [e:t] is a present-stage expression, whereas in $[e]_n^1$, it is a n+1-stage expression. This post-processing (optimized in version N102) was employed in BER MetaOCaml until the present version N114.

The translation [e:t] is the identity, until it comes to bracket:

$$\begin{bmatrix} i : int \end{bmatrix} = i : int \quad \begin{bmatrix} x^0 : t \end{bmatrix} = x : t \quad \begin{bmatrix} (e \ e') : t \end{bmatrix} = (\begin{bmatrix} e \end{bmatrix} \begin{bmatrix} e' \end{bmatrix}) : t$$
$$\begin{bmatrix} (\lambda x^0 : t' \cdot e : t) : t' \to t \end{bmatrix} = (\lambda x : t' \cdot \begin{bmatrix} e : t \end{bmatrix}) : t' \to t$$

Switch-over:

Γ

$$\lceil \langle e:t \rangle \rceil = \lceil e:t \rceil_0^1 : \langle t \rangle \qquad \lceil \sim (e:\langle t \rangle) \rceil_0^1 = \lceil e:\langle t \rangle \rceil$$

A future-stage translation:

$$\begin{bmatrix} i : \operatorname{int} \end{bmatrix}_{n}^{1} = \operatorname{lift}_{\operatorname{int}} i : \langle \operatorname{int} \rangle$$

$$\begin{bmatrix} x^{m+1} : t \end{bmatrix}_{n}^{1} = x : \langle t \rangle (m \le n)$$

$$\begin{bmatrix} x^{0} : t \end{bmatrix}_{n}^{1} = \begin{cases} \operatorname{lift}_{t} x^{n} : \langle t \rangle & \text{if } x \in \Gamma_{i} \\ \operatorname{lift}_{t} x : \langle t \rangle & \text{otherwise} \end{cases}$$

$$\begin{bmatrix} (e \ e') : t \end{bmatrix}_{n}^{1} = \operatorname{mka} \left[e \end{bmatrix}_{n}^{1} \left[e' \right]_{n}^{1} : \langle t \rangle$$

$$\begin{bmatrix} \langle x^{n+1} : t' \cdot e : t \end{bmatrix}_{n}^{1} = \operatorname{mkl} \left(\lambda x : \langle t' \rangle \cdot \left[e : t \right]_{n}^{1} \right) : \langle t' \to t \rangle$$

$$\begin{bmatrix} \langle e : t \rangle \end{bmatrix}_{n}^{1} = \operatorname{mkbr} \left[e : t \end{bmatrix}_{n+1}^{1} : \langle t \rangle$$

$$\begin{bmatrix} \langle e : \langle t \rangle \end{bmatrix}_{n+1}^{1} = \operatorname{mks} \left[e : \langle t \rangle \end{bmatrix}_{n}^{1} : \langle t \rangle$$

Figure 3. Code-generating combinators

Figure 3 lists the code-generating functions: the producers of values of the code type.¹ Here lift_t is the family indexed by type t.²

Type preservation of the translation does not seem obvious: after all, identifiers at any future stage are translated as present-stage identifiers, of the same name but at the *changed type*: *<t>*, which is furthermore independent of stage. Likewise, functions at a future stage are translated into present-stage functions, but at a different type. We discuss the formal properties in in the next section. At present we note that all translation equations satisfy (1).

¹In MetaOCaml, they are called lift_constant_int, ..., build_fun, build_apply, etc.
²If such lifting functions exist for all types and how they can be implemented is a fascinating question that we do not have space to answer.

For example, the (specialized) eta from §1:

$$f^0: < int > \rightarrow < int > . < \lambda x^1: int. ~ (f < x >)$$

of the type (<int> \rightarrow <int>) \rightarrow <int \rightarrow int> is translated to the Base1 expression

$$\lambda f:\langle \text{int} \rangle \rightarrow \langle \text{int} \rangle$$
. mkl $\lambda x:\langle \text{int} \rangle$. f x

clearly of the same type. The origin of the name eta should be also clear.

As a more interesting example, consider

 $\langle \lambda x^1$: int. $\langle \lambda y^2$: int. $x + y \rangle$: $\langle \text{int} \rightarrow \text{int} \rangle$

which has two CSPs, both appearing at stage 2: one is x, defined at stage 1, and the other is addition, defined in the initial environment, at stage 0. The translated Base₁ expression is:

 $\operatorname{mkl} \lambda x : <\operatorname{int} > \operatorname{mkbr} (\operatorname{mkl} \lambda y : <\operatorname{int} > \operatorname{mka} (\operatorname{mka} (\operatorname{mkid} "+") x) y)$

It has the same $\langle int \rightarrow \langle int \rangle$ type, as one can easily verify.

3.1 Optimized translation

A careful look at the translation rules just presented shows many opportunities for optimization. First of all, since $\lceil - \rceil$ is mostly the identity, it is tempting to cut it out and hence eliminate the useless traversing and rebuilding of the Typedtree. Furthermore, $\lceil - \rceil_n^1$ does not essentially use *n* and can be simplified.

We now present the optimized translation. To avoid $\lceil -\rceil$ it requires the integration with the type checker. In principle, we can combine the translation and the type reconstruction completely. For example, the type reconstruction judgments for integer literals would then become:

 $\Gamma \vdash_0 i \Rightarrow i : int \qquad \Gamma \vdash_{n+1} i \Rightarrow \text{lift}_{int} i : < \text{int} >$

That would unwise, however: we have to effectively duplicate the type checking rules, for stage 0 and stage > 0. A better idea is to leave the stage-invariant rules (which is most of them) as they are and introduce a selective translation $\lfloor e : t \rfloor$, defined as the simplified $\lceil e : t \rceil_0^1$, to wit:³

$$\lfloor i : \operatorname{int} \rfloor = \operatorname{lift}_{\operatorname{int}} i : <\operatorname{int} >$$

$$\lfloor x^{m+1} : t \rfloor = x :$$

$$\lfloor x^0 : t \rfloor = \begin{cases} \operatorname{mkid}_t "x" : & \operatorname{if} x \in \Gamma_l \\ \operatorname{lift} x : & \operatorname{otherwise} \end{cases}$$

$$\lfloor (e \ e') : t \rfloor = \operatorname{mka} \lfloor e \rfloor \lfloor e' \rfloor :$$

$$\lfloor \lambda x^{n+1} : t' \cdot e : t \rfloor = \operatorname{mkl} (\lambda x : \lfloor e : t \rfloor) :

$$\lfloor \sim (e :) \rfloor = e :$$$$

The typing judgment is now $\Gamma \vdash_n e \Rightarrow e' : t$ where *e* is an (un-annotated) expression of the Staged calculus and *e'* is the type-annotated expression of Base₁ extended with $\sim e$ and stage-annotated variables. (Bindings in Γ are also stage-annotated. For present stage, the annotation may be dropped.) Such extended calculus is called Base₂. Quite unexpectedly, Base₂ has no need for brackets; it only needs escapes, hence the changes to the OCaml Typedtree are minimal. In fact, there are no changes at all, thanks to Typedtree attributes: an escape is indicated by a dedicated attribute attached to a Typedtree node.

Figure 4 presents the pseudo-code of the optimized translation integrated with type reconstruction. The figure makes it clear how

 $\begin{array}{c} \overline{\Gamma \vdash_n i \Rightarrow i: \mathrm{int}} & \frac{x^m: t \in \Gamma}{\Gamma \vdash_n x \Rightarrow x^m: t} \ m \leq n \\ \\ \overline{\Gamma \vdash_n i \Rightarrow i: \mathrm{int}} & \frac{\Gamma \vdash_n e \Rightarrow x^m: t}{\Gamma \vdash_n e \Rightarrow x^m: t} \ m \leq n \\ \\ \overline{\Gamma \vdash_n e \Rightarrow e: t' \rightarrow t} & \Gamma \vdash_n e' \Rightarrow e': t' \\ \hline \overline{\Gamma \vdash_n e e' \Rightarrow (e: (t' \rightarrow t) e': t'): t} \\ \\ \overline{\Gamma \vdash_n \lambda x. e \Rightarrow (\lambda x^n: t' \cdot e \Rightarrow e: t)} \\ \\ \overline{\Gamma \vdash_n e \Rightarrow e: t} & \Gamma \vdash_{n+2} e \Rightarrow e: t \\ \hline \overline{\Gamma \vdash_n e \Rightarrow e: t} & \Gamma \vdash_{n+1} e \Rightarrow e: t \\ \hline \Gamma \vdash_n e \Rightarrow e: dx \\ \hline \Gamma \vdash_n e \Rightarrow e: dx \\ \hline \Gamma \vdash_{n+1} e \Rightarrow e: dx \\ \hline \Gamma \vdash_{n+2} e \Rightarrow e' e' = dx \\ \hline \Gamma \vdash_{n+1} e \Rightarrow e' = dx \\ \hline \Gamma \vdash_{n+2} e \Rightarrow e' = dx \\ \hline \Gamma \vdash_{n+2} e \Rightarrow e' = dx \\ \hline \Gamma \vdash_{n+2} e \Rightarrow e' = dx \\ \hline \Gamma \vdash_{n+2} e \Rightarrow e' = dx \\ \hline \Gamma \vdash_{n+2} e \Rightarrow e' = dx \\ \hline \Gamma \vdash_{n+2} e \Rightarrow e' = dx \\ \hline \Gamma \vdash_{n+2} e' = dx \\ \hline \Gamma$

Figure 4. Type-checking and translation of Staged into Base₂.

the Base type reconstruction – that is, the Typedtree construction in the ordinary OCaml – has to be modified for staging. Most of the rules (see constant and application rules) are unmodified. We still need to maintain the stage (as a global mutable variable in the current implementation). The rule for lambda (and other binding forms) has to annotate the bound variable with its stage as it is put into the environment. We do it by adding an attribute bearing the stage to the value_description of the variable. The variable rule has to check that the stage of the variable is less than or equal the current stage, and to put the stage-annotated variable into Typedtree. In the implementation, nothing needs to be done for the latter: The Texp_ident node of the Typedtree carries the value_description taken from the environment, which already has the stage attribute. The only significant changes are the rules for brackets and escapes (represented in Parsetree as extension nodes).

The selective translation $\lfloor - \rfloor$ is indeed done only on the parts of the overall Typedtree that represent future-stage sub-expressions. Therefore, when compiling plain OCaml programs, MetaOCaml imposes *no* overhead.

Proposition. If $\Gamma \vdash_n e \Rightarrow e : t$ in the Staged calculus then $\Gamma \vdash_n e \Rightarrow e' : t$ in the optimized translation.

Proposition. If $\Gamma \vdash_n e \Rightarrow e' : t$, then e' has no nested escapes.

Corollary. If $\Gamma_i \vdash_0 e \Rightarrow e' : t$ than e' is strictly a Base₁ expression: it contains no escape nodes or stage-annotated bindings. The type reconstruction hence gives the ordinary OCaml Typedtree, which can then be processed by the OCaml back end as is.

Theorem. If $\Gamma \vdash_0 e \Rightarrow e' : t$ then $\Gamma \vdash e' \Rightarrow e' : t$ in Base₁ where e' is e' with all type annotations removed.

4 Related work

The idea of implementing code templates by a translation into code combinators can be traced back to Lisp: quasi-quotes are commonly implemented as macros, expanding into S-expression combinators (cons and list).

The translation $\lceil - \rceil$ and $\lceil - \rceil_n^1$ was implemented in BER Meta-OCaml N101 described in [4]. The translation however was not presented formally. It is similar to [1, Figure 3]. However, our translation is typed. Mainly, we use code combinators instead of data types and may hence keep the code representation abstract. The

³performed by trx_translate of typing/trx.ml

The optimized translation in §3.1 is novel: the present paper is the first presentation of it – and BER MetaOCaml N114 is the first implementation.

5 Need multiple stages?

One may have noticed that the eta-generator-generator etah in §1 was rather contrived. That is no accident: there are hardly any realistic examples of needing more than one future stage. This has been noticed before. In his retrospective [6], Sheard writes: "There is no limit to the number of stages in a MetaML program. This has been useful theoretically, but has found very little practical use. Programmers find it hard to write programs with more than a few stages." [6, §21]

The only somewhat realistic case I am aware of is generating code that includes a run-time specializer (evoking just-in-time compilation): for example, generating code for power n x simultaneously with the code to specialize the power function to a specific value of n, and the overall driver that switches to the specialized version if power n x was invoked for a specific n often enough. (This example was suggested by Sven Bodo Sholz.) Even then, such an example seems better implemented using the tagless-final approach coupled with one-future-stage staging.

I would like to ask the readers if there is a value in continuing to maintain the ability to nest brackets arbitrarily. If not, it would make sense to limit the bracket nesting to the single level, which notably simplifies the implementation.⁴

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⁴One may still build generators of generators, using CSP.